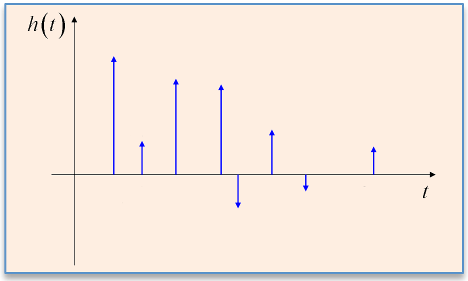
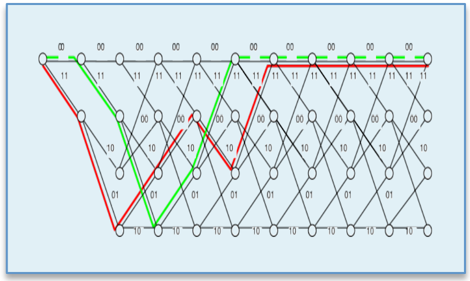
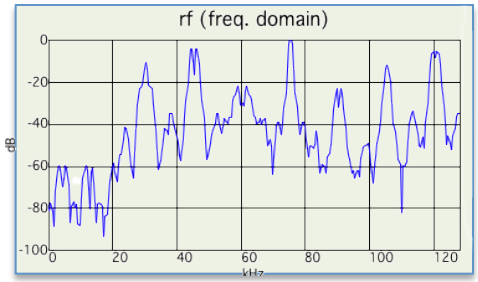
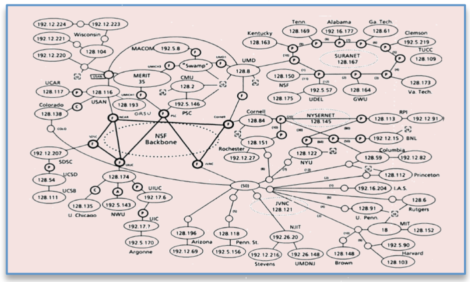








INTRODUCTION TO EECS II

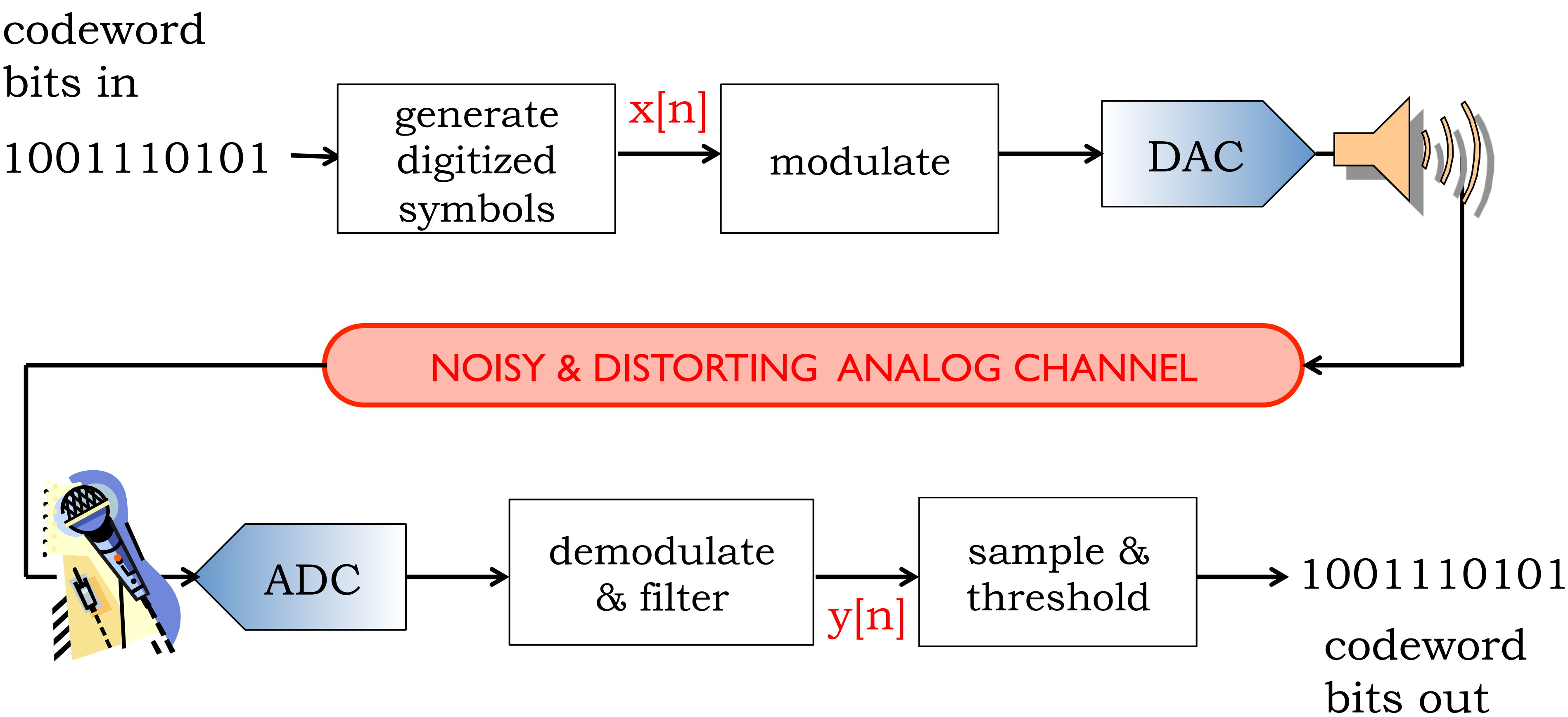
DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2014
Lecture #10

- Modulation/demodulation
- Channel models
- Linear time-invariant (LTI) models
- Superposition

Back to noise-free distortion ···

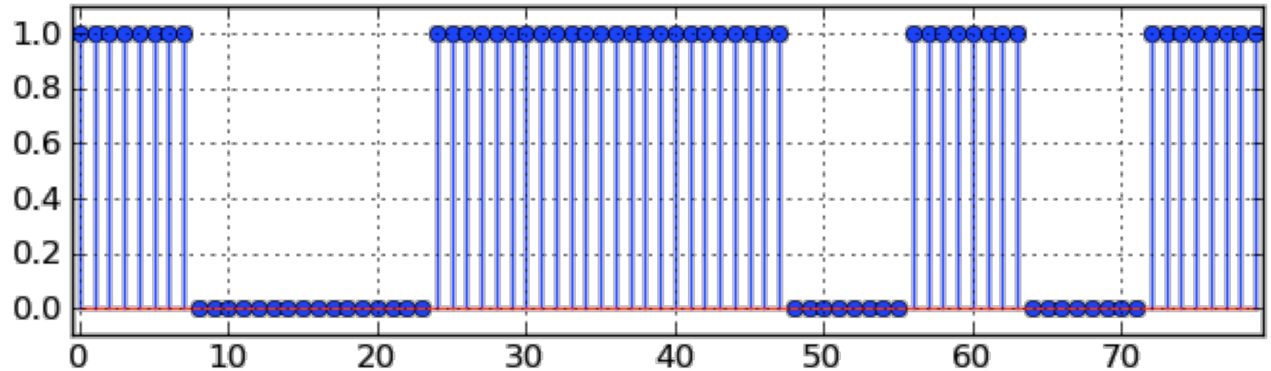
From Baseband to Modulated Signal, and Back



Digitized Symbols, x[n]

1 0 0 1 1 1 0 1 0 1

Sample value



Sample number n
(discrete-time index)

Modulation (at the Transmitter)

Adapts the digitized signal x[n] to the characteristics of the channel.

e.g., Acoustic channel from laptop speaker to microphone is *not* well suited to transmitting *constant* levels v_0 and v_1 to represent 0 and 1. So instead transmit **sinusoidal** pressure-wave signals proportional to speaker voltages

$$v_0 \cos(2\pi f_c t) \quad \text{and} \quad v_1 \cos(2\pi f_c t)$$

where f_c is the *carrier frequency* (e.g., 2kHz; wavelength at 340 m/s = 17cm, comparable with speaker dimensions) and

$$v_0 = 0 \qquad v_1 = V > 0$$

(on-off or *amplitude* keying)

or alternatively

$$v_0 = -V \qquad v_1 = V > 0$$

(bipolar or *phase-shift* keying)

Could also key the *frequency*.

From Brant Rock tower, radio age was sparked

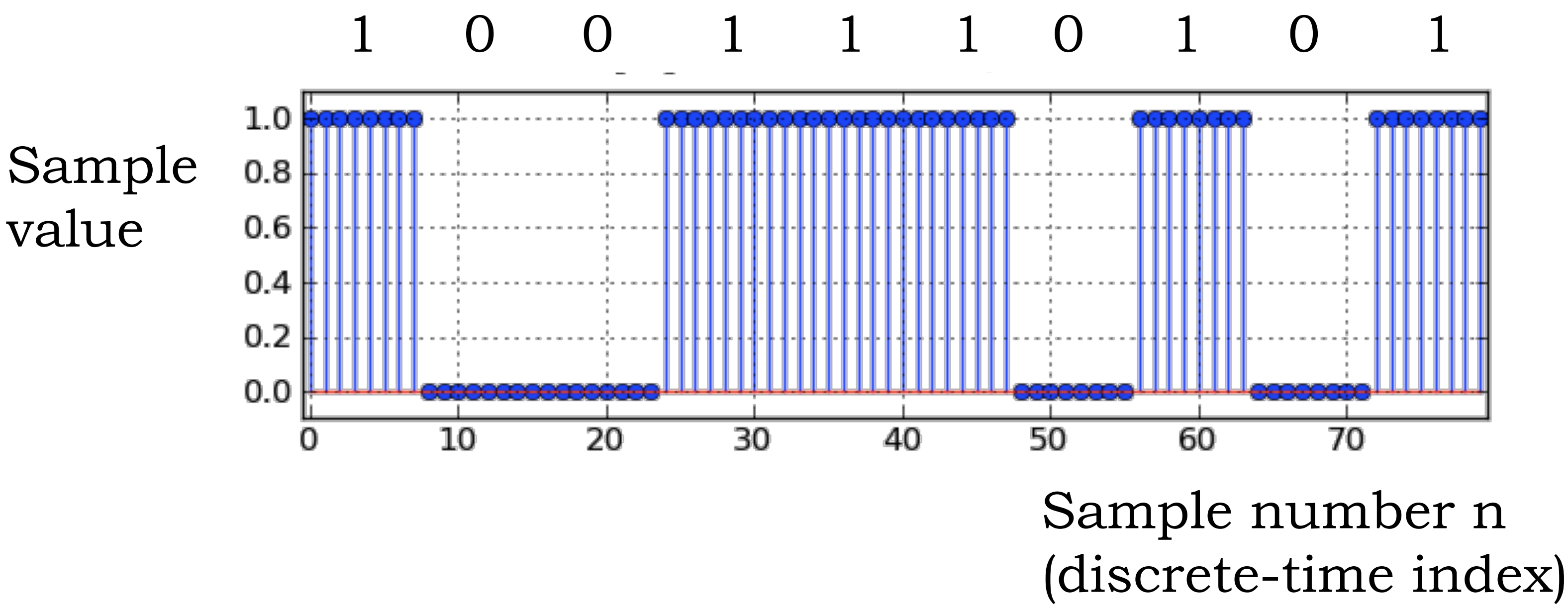
By Carolyn Y. Johnson, Globe Staff | July 30, 2006

MARSHFIELD, MA -- A century ago, radio pioneer Reginald A. Fessenden* used a massive 420-foot radio tower that dwarfed Brant Rock to send voice and music to ships along the Atlantic coast, in what has become known as the world's first voice radio broadcast. This week, Marshfield will lay claim to its little-known radio heritage with a three-day extravaganza to celebrate the feat -- including pilgrimages to the base of the long-dismantled tower, a cocktail to be named the Fessenden Fizz, and a dramatic reenactment of the historic moment, called ``Miracle at Brant Rock."

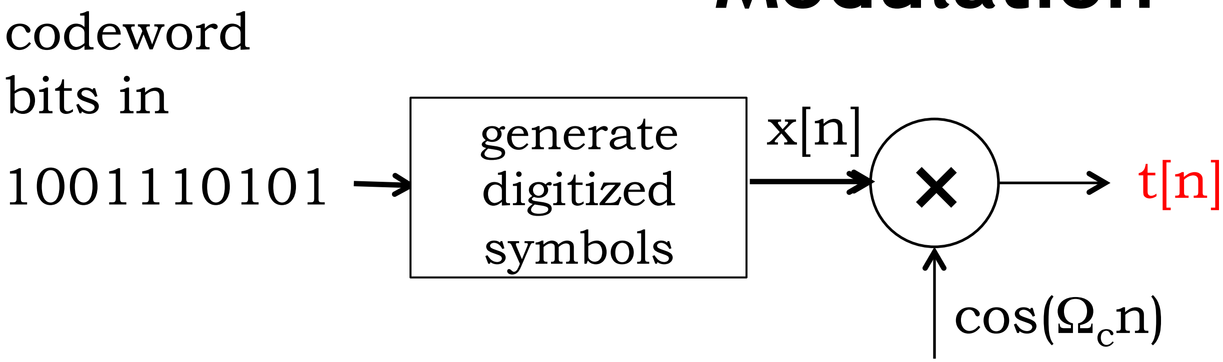
Amplitude Modulation (AM)


Wireless Station, Brant Rock, Mass.

Digitized Symbols

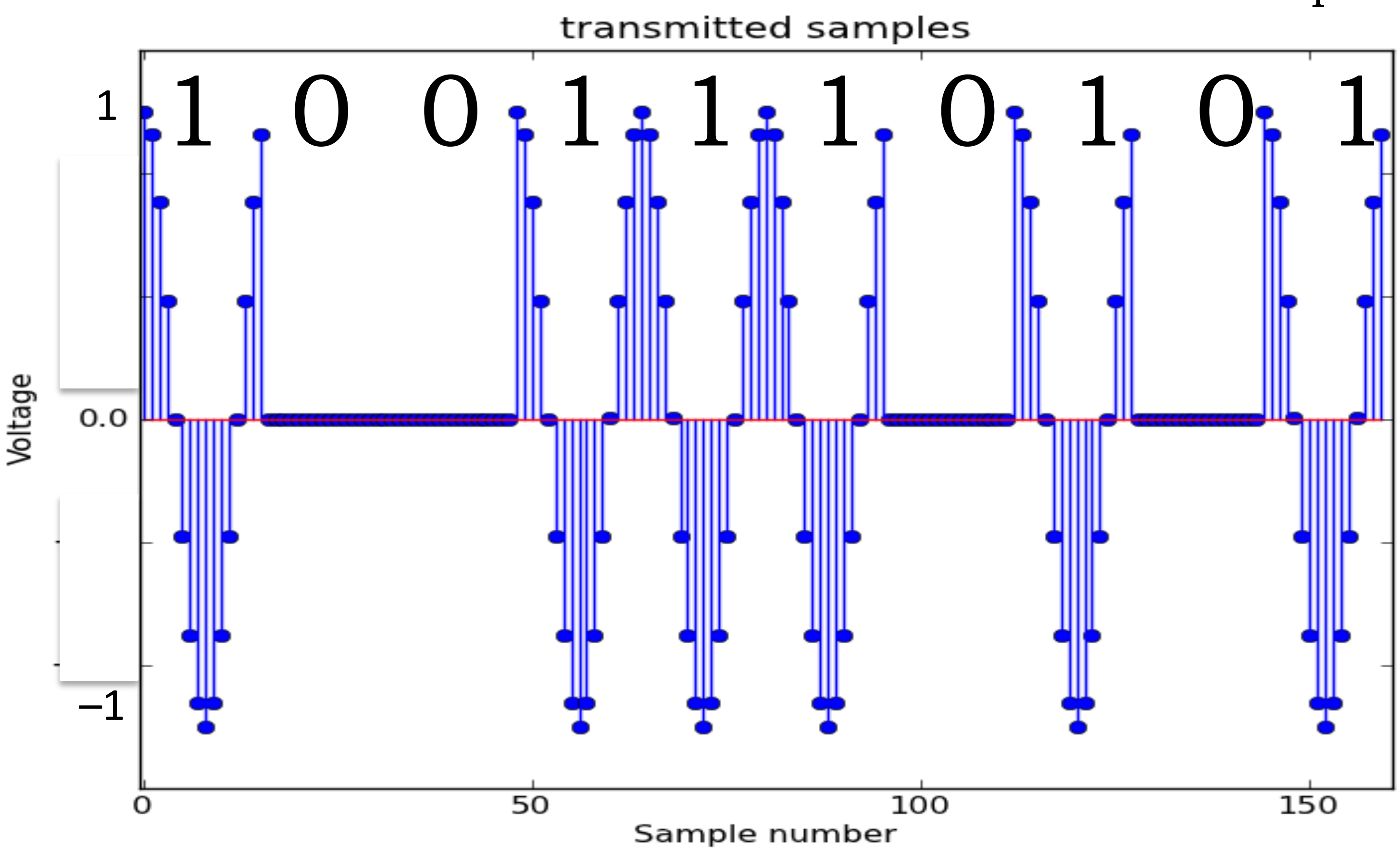


Modulation



$\Omega_c = 2\pi/16$

16 samples per cycle



Ideas for Demodulation

- For on-off keying, it suffices to detect when there's signal and when there isn't, since we're only trying to distinguish

$$v_0 = 0 \qquad v_1 = V > 0$$

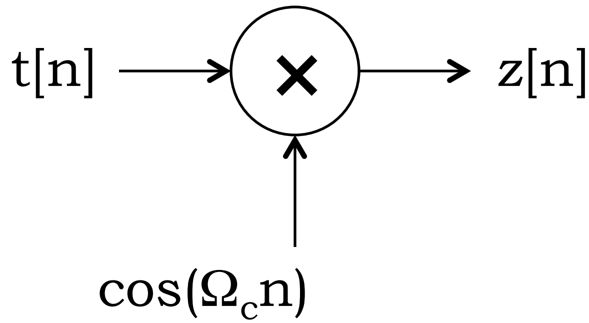
Many ways to do that, e.g., take absolute value and then local average over half-period of carrier

- For bipolar keying, we need the sign:

$$v_0 = -V \qquad v_1 = V > 0$$

Demodulation

Assuming no distortion or noise on channel, so what was transmitted is received



Why not just divide t[n] by cos(Ω_cn) ?

$$z[n] = t[n]\cos(\Omega_c n)$$

$$z[n] = x[n]\cos(\Omega_c n)\cos(\Omega_c n)$$

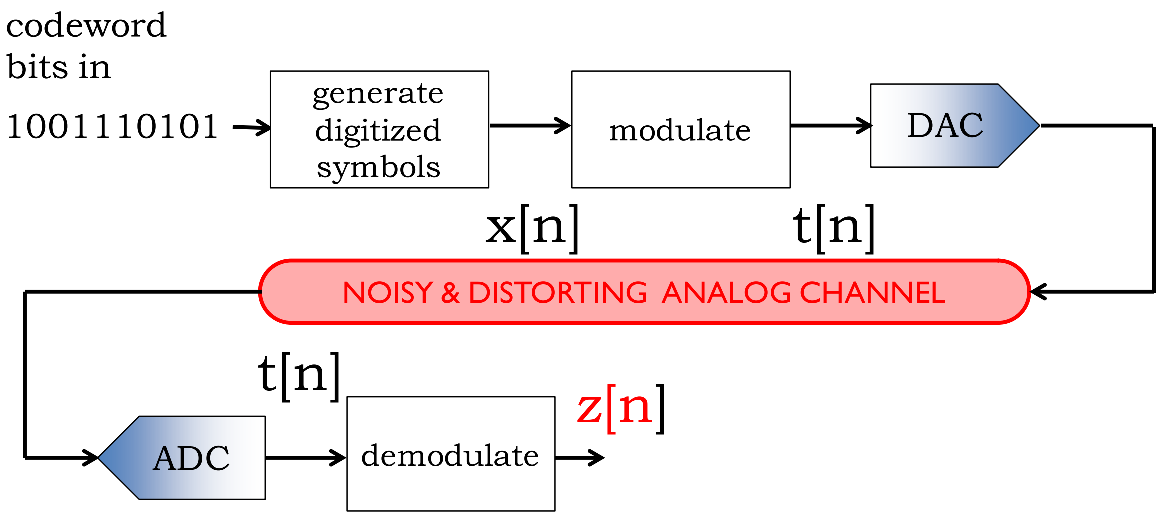
$$z[n] = 0.5x[n](1+\cos(2\Omega_c n))$$

'Heterodyning' - invented by Fessenden

$$z[n] = 0.5x[n] + 0.5x[n]\cos(2\Omega_c n)$$

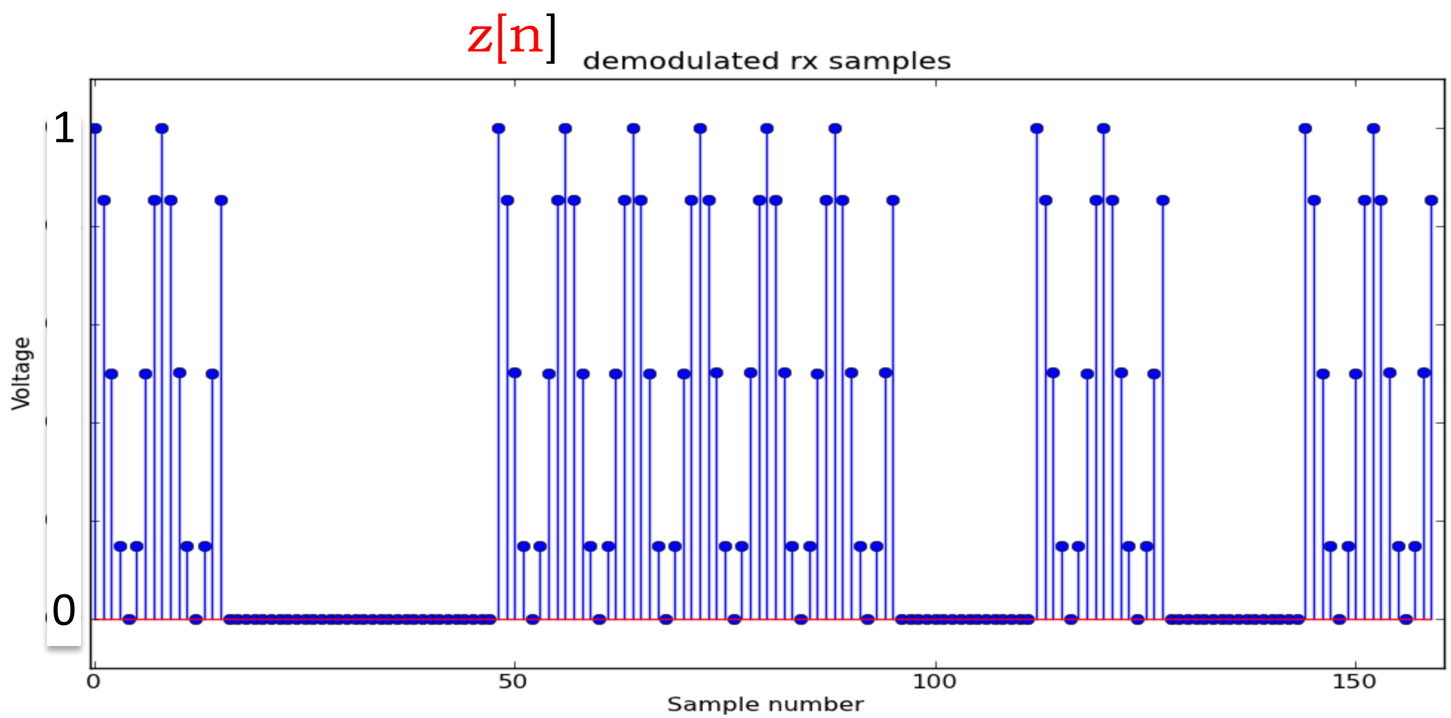
Extract this!

Demodulation

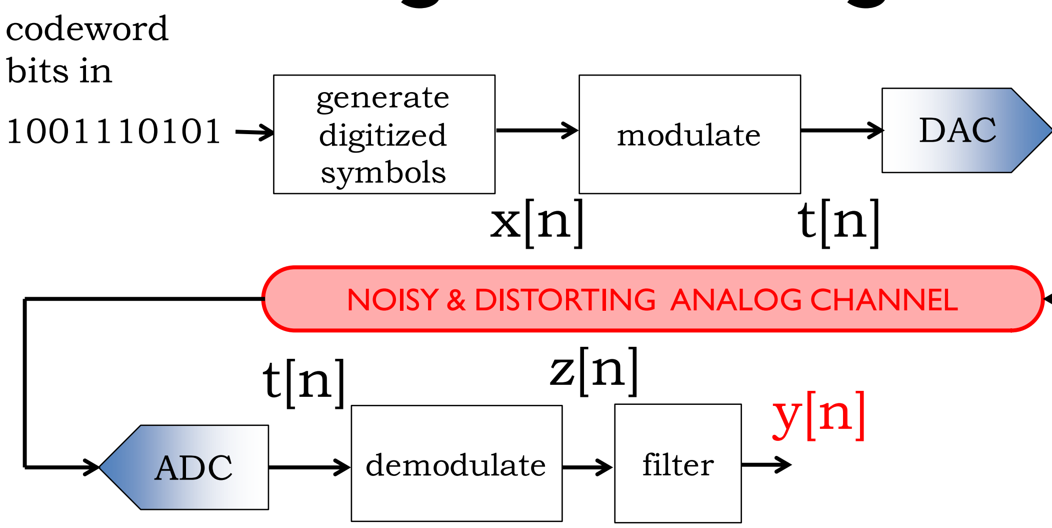


$\Omega_c = 2\pi / 16$

16 samples per cycle

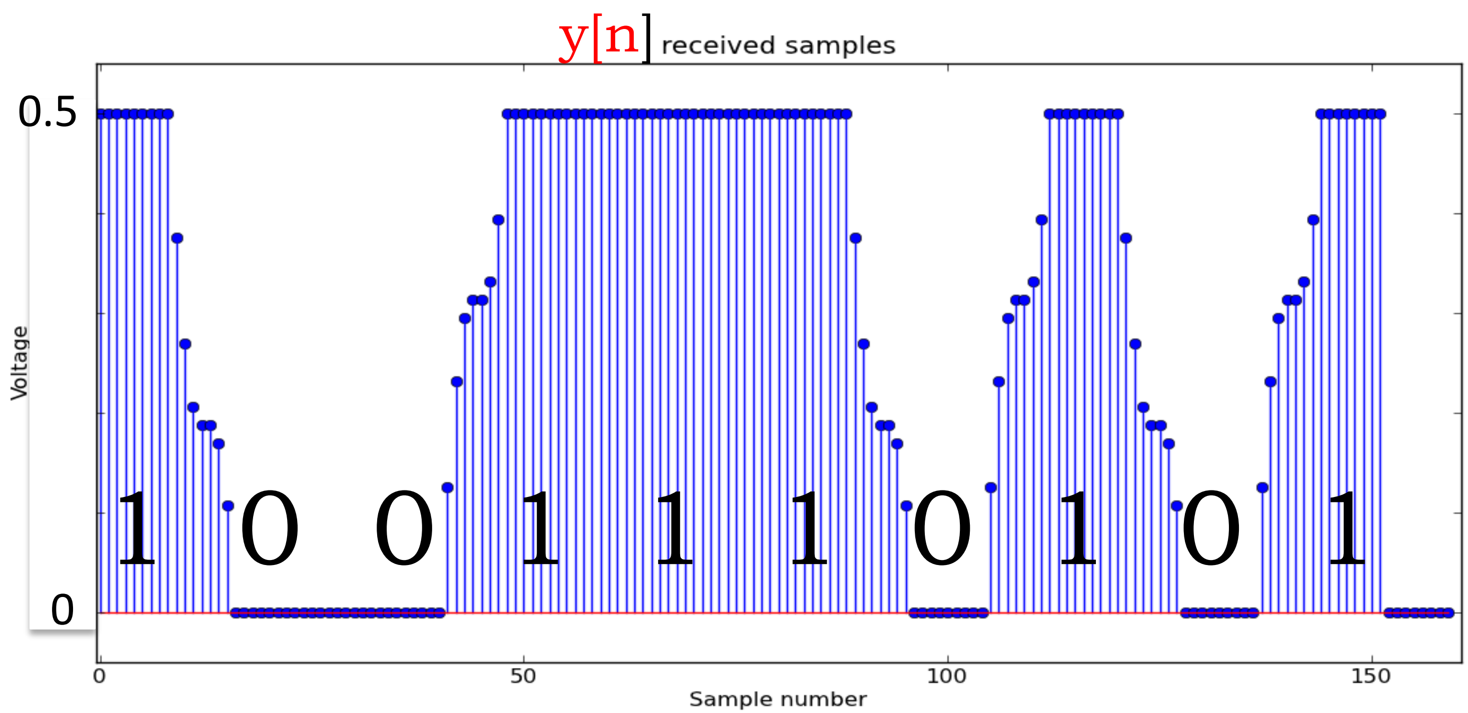


Filtering: Removing the $2\Omega_c$ component

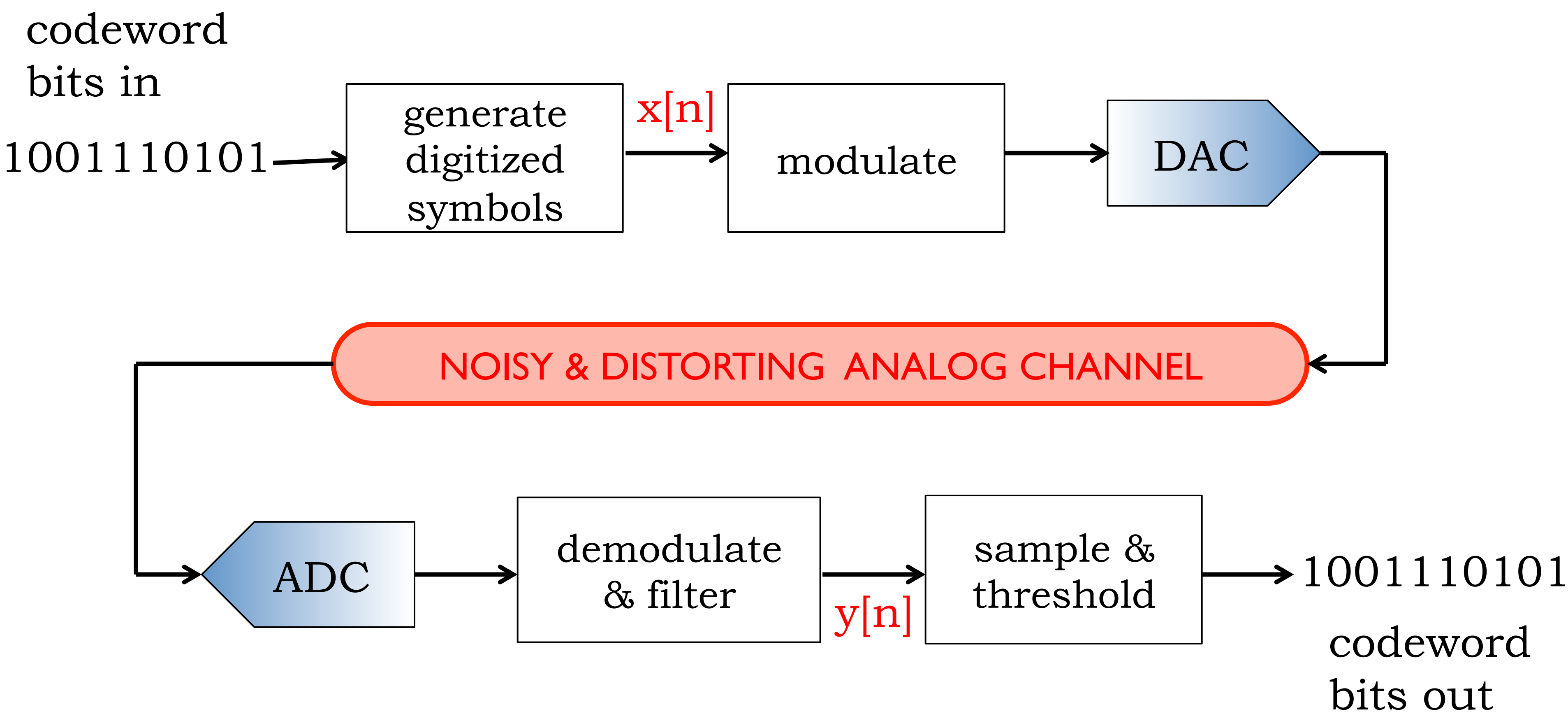


$\Omega_c = 2\pi/16$

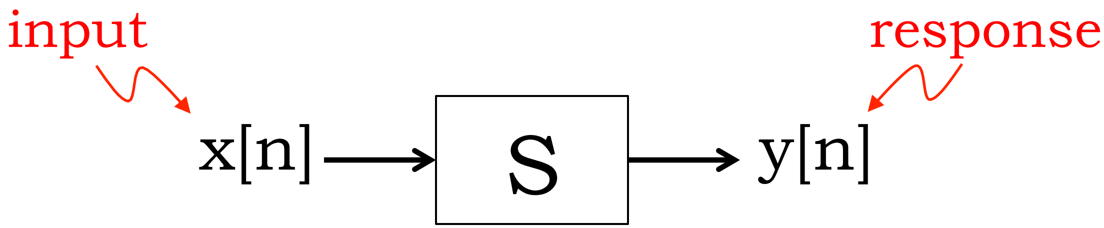
16 samples per cycle



Modeling The Baseband Channel



The Baseband** Channel



The sequence of *output* values y[.] is the *response* of system S to the *input* sequence x[.]. We assume y[n] is uniquely determined by all of x[.], i.e., x[k] for all k (nothing else needs to be known).
The above picture is a snapshot at a particular time n.

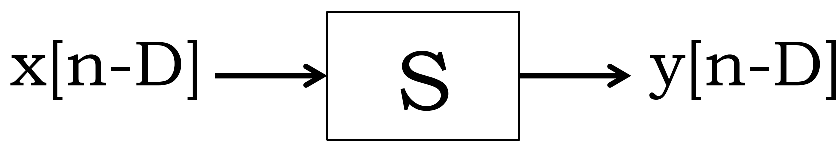
The system is causal if y[k] depends only on x[j] for j≤k

**From before the modulator till after the demodulator & filter

Time Invariant Systems

Let y[.] be the response of S to input x[.].

If for all possible sequences x[n] (with n ranging over all integer values) and an arbitrary integer D:



then system S is said to be *time invariant* (TI).

A time shift D in the input sequence to S results in an identical time shift of the output sequence.

Linear Systems

Let $y_1[.]$ be the response of S to an arbitrary input $x_1[.]$ and $y_2[.]$ be the response to an arbitrary $x_2[.]$.

If, for arbitrary scalar coefficients *a* and *b*, we have:

$$ax_1[n]+bx_2[n] \longrightarrow \boxed{\ S\ } \longrightarrow ay_1[n]+by_2[n]$$

then system S is said to be *linear*. If the input is the weighted sum of several signals, the response is the *superposition* (i.e., same weighted sum) of the response to those signals.

One key consequence: If the input is identically 0 for a linear system, the output must also be identically 0.

Linear and Time Invariant (LTI) System/Model

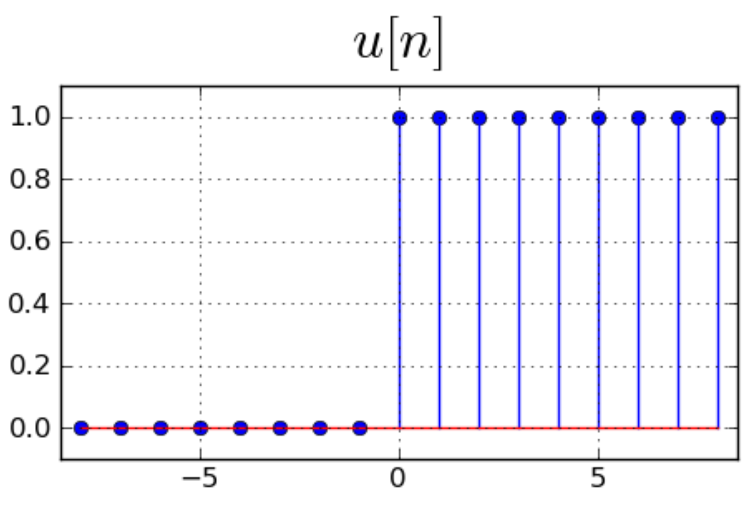
… is (1) linear, and (2) time invariant!

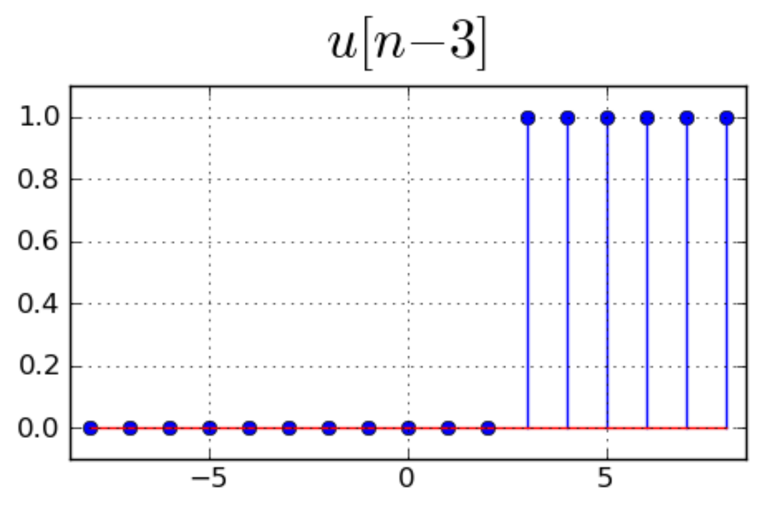
Assume LTI models from now on, for 6.02.

Unit Step

A simple but useful discrete-time signal is the *unit step* signal or function, u[n], defined as

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

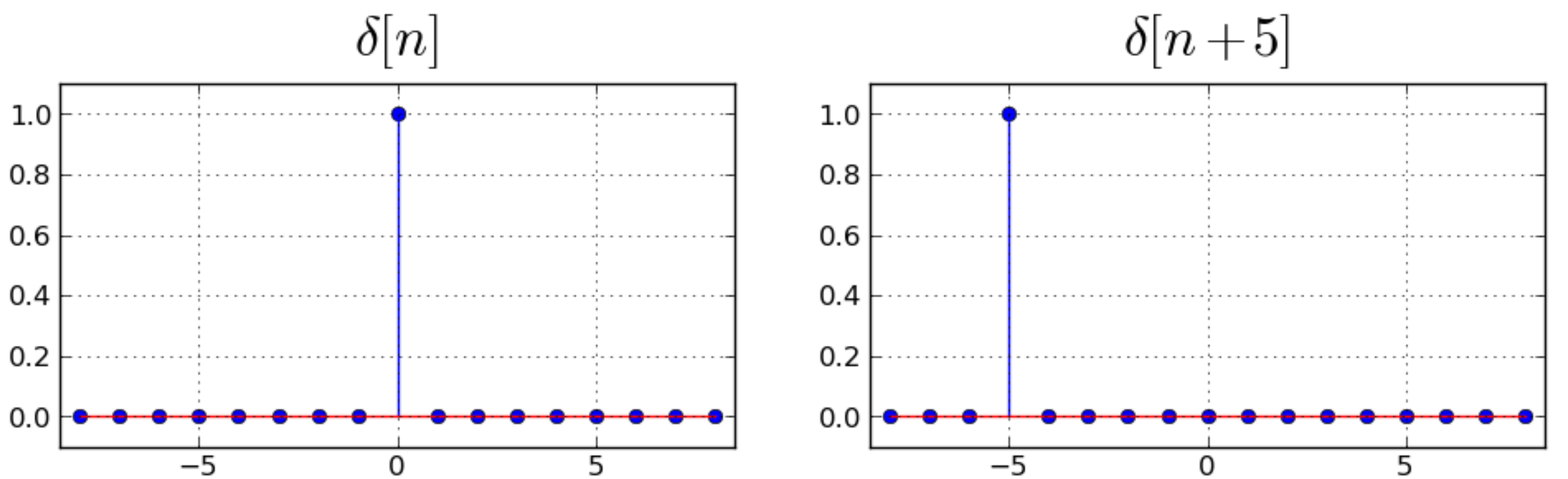




Unit Sample

Another simple but useful discrete-time signal is the *unit sample* signal or function, δ[n], defined as

$$\delta[n] = \boxed{u[n] - u[n-1]} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



Note that standard algebraic operations on signals (e.g. subtraction, addition, scaling by a constant) are defined in the obvious way, instant by instant.

Unit Sample and Unit Step Responses

Unit sample   Unit sample response

$$\delta[n] \longrightarrow \boxed{S} \longrightarrow h[n]$$

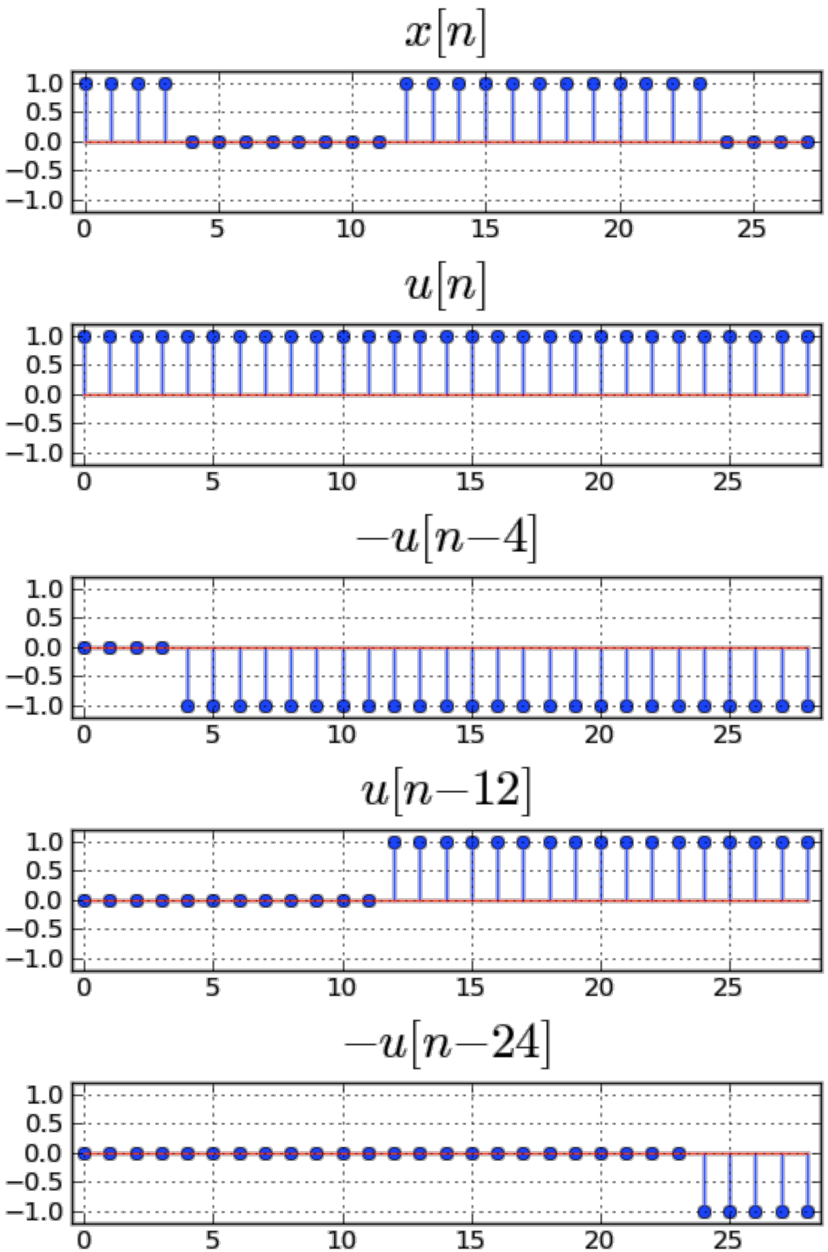
The *unit sample response* of a system S is the response of the system to the unit sample input. We will always denote the unit sample response as h[n].

For a causal linear system, $h[n] = 0$ for $n < 0$.

Similarly, the *unit step response* s[n]:

Unit step   Unit step response

$$u[n] \longrightarrow \boxed{S} \longrightarrow s[n]$$



Unit Step Decomposition

"Rectangular-wave" digital signaling waveforms, of the sort we have been considering, are easily decomposed into time-shifted, scaled unit steps --- each transition corresponds to another shifted, scaled unit step.

e.g., if x[n] is the transmission of 1001110 using 4 samples/bit:

$$x[n] = u[n] - u[n-4] + u[n-12] - u[n-24]$$

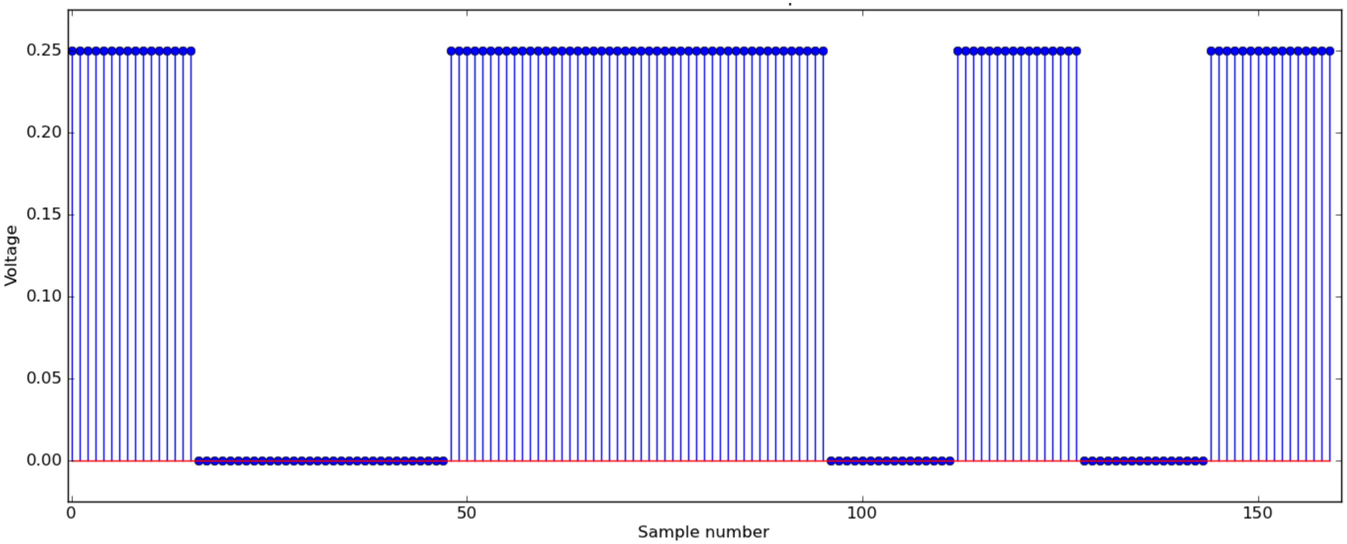
… so the corresponding response is

$$x[n] = u[n] - u[n-4] + u[n-12] - u[n-24]$$

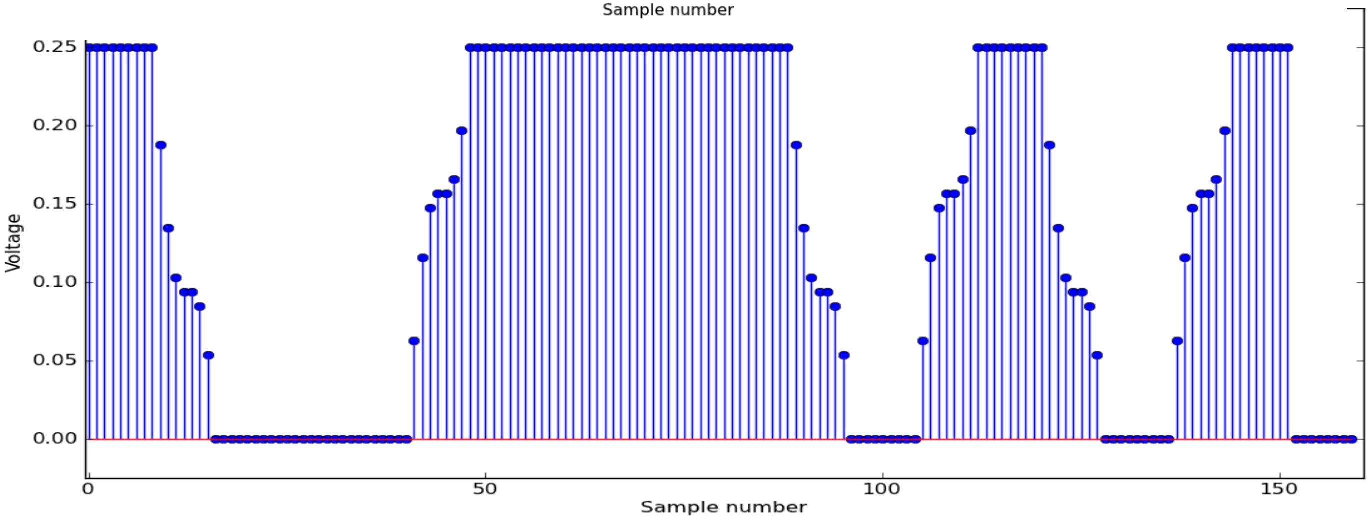
$$y[n] = s[n] - s[n-4] + s[n-12] - s[n-24]$$

Note how we have invoked linearity and time invariance!

Example

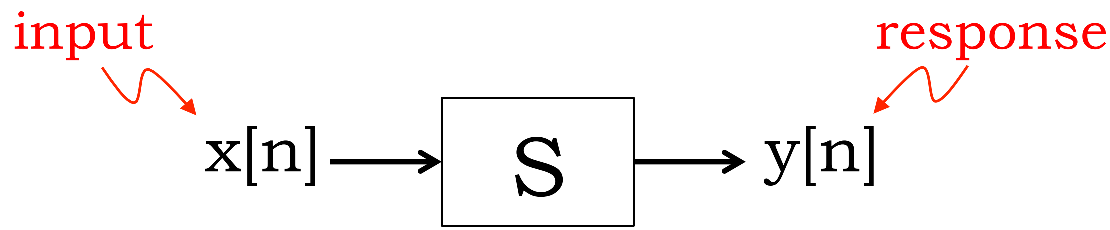


Voltage
0.25
0.20
0.15
0.10
0.05
0.00
0
50
100
150
Sample number

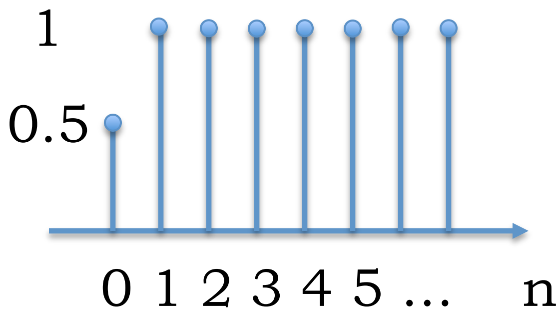


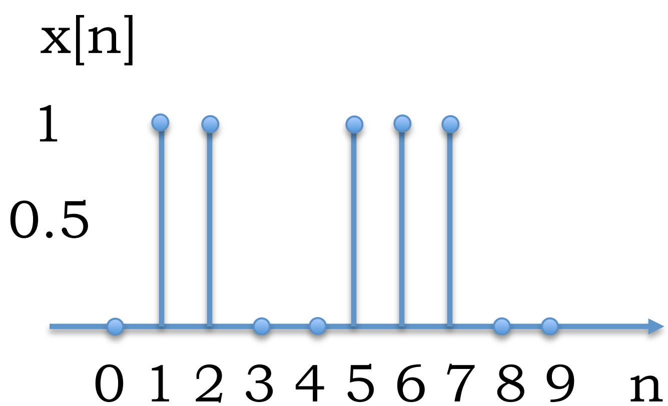
Voltage
0.25
0.20
0.15
0.10
0.05
0.00
0
50
100
150
Sample number

Spot Quiz



Unit step response: s[n]





Find y[n]:

1. Write x[n] as a function of unit steps

2. Write y[n] as a function of unit step responses

3. Draw y[n]